

Hadronic Lorentz Violation in Chiral Perturbation Theory

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Abstract

Any possible Lorentz violation in the hadron sector must be tied to Lorentz violation at the underlying quark level. The relationships between the theories at these two levels are studied using chiral perturbation theory. Starting from a two-flavor quark theory that includes dimension-four Lorentz-violation operators, the effective Lagrangians are derived for both pions and nucleons, with novel terms appearing in both sectors. Since the Lorentz violation coefficients for nucleons and pions are all related to a single set of underlying quark coefficients, it is possible to place approximate bounds on pion Lorentz violation using only proton and neutron observations. The resulting bounds on four pion parameters are at the 10^{-23} level, representing improvements of ten orders of magnitude.

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1 Introduction

Over the last twenty years, there has been a tremendous surge of interest in the possibility that the fundamental Lorentz and CPT symmetries might actually be violated in nature. Although no such symmetry violations have yet been discovered experimentally, they might be part of a fundamental theory of quantum gravity. In fact, if a violation of Lorentz symmetry is ever discovered, it will be proof of new fundamental physics beyond the standard model and general relativity, and the discovery will provide important information about the nature of the new physics.

It has been recognized for a long time that these factors make precise tests of Lorentz symmetry very important. More recently, however, interest in Lorentz symmetry violation has grown because of the development of an effective field theory, known as the standard model extension (SME), that can be used to describe all forms of Lorentz violation that may exist in a quantum field theory built around the standard model fields [1, 2]. The SME is quite general, and even the minimal SME (mSME)—which is restricted to contain only gauge invariant, superficially renormalizable operators in its action—includes many more forms of Lorentz violation than previous studies had ever looked at. The ability of the SME to parameterize such a wide array of Lorentz-violating phenomena has led to a tremendous expansion in experimental tests of Lorentz symmetry—in practically all sectors of the theory. An up-to-date summary of the results of these tests may be found in [3].

However, understanding of the SME is still far from complete. The SME is formulated as a relativistic field theory, in terms of the fundamental quark, lepton, gauge, and Higgs fields of the standard model, and the relationships between the parameters in the fundamental SME Lagrangian and experimental observables can be complicated. The most important outstanding issue in this area arises from the fact that at low energies, the standard model's strongly interacting degrees of freedom are composite hadrons. There are many extremely precise constraints on effective Lorentz violation coefficients for protons and neutrons, as well as weaker constraints for other hadrons. However, it has not been possible to translate these constraints into bounds on the more basic parameters appearing in the SME action.

Using chiral perturbation theory (χ PT) [4, 5, 6] (also see Ref. [7] for a pedagogical introduction), we shall examine the relationships between quark- and hadron-level parameterizations of Lorentz violation. (Another recent χ PT analysis [8] has looked at a quite different set of terms from the ones we shall be considering.) In section 2, we introduce Lorentz violation for the fundamental quark fields. Then, in section 3, we construct the χ PT action in two-flavor QCD for both pions and nucleons, at leading order (LO) in each sector. This will reveal which Lorentz-violating parameters in the quark sector contribute to particular operators built out of the hadron fields. We shall find that even at the lowest chiral orders, there are expected to be terms in the hadronic theory that have previously not been studied. Using the relationships we have uncovered between the actions in different sectors, we shall look at how bounds on the Lorentz-violating behavior of one type of hadron may be used to place constraints on different phenomena involving entirely different species of particles in section 4. Finally, section 5 summarizes our main results and the outlook for future work.

2 Lorentz Violation at the Quark Level

The basic idea behind the SME is to look at an effective field theory containing operators constructed out of the standard model fields, but which does not respect Lorentz symmetry. The novel operators can have arbitrary free Lorentz indices. These indices are contracted with tensor-valued coefficients, which serve as preferred vector or tensor backgrounds in spacetime. Otherwise identical experiments done in different reference frames may yield different outcomes because of these background tensors. The vector and tensor backgrounds may be constrained by comparing the results of experiments with the apparatus in different orientations relative to the fixed stars, or experimental setups moving with different velocities.

There are now many strong constraints on the Lorentz-violating parameters of the mSME, coming from experiments in atomic, nuclear, and astroparticle physics. Any local, stable field theory that violates CPT symmetry also violates Lorentz invariance [9]. So the SME is also the unique well-behaved effective field theory governing CPT violation, and the mSME is also used for parameterizing constraints on CPT violation. In most (but not all) cases, Lorentz-violating operators with odd numbers of free Lorentz indices are CPT odd, while those with even number of indices are CPT even. In this paper, we shall only be considering CPT-even operators with two free Lorentz indices.

We are also restricting our attention to the quark sector of the mSME, so the possible Lorentz-violating operators will be built out of quark field bilinears. These are the fundamental operators of the effective field theory. However, it is conventional when studying hadronic systems to consider similar Lorentz-violating operators for the composite quanta—protons, neutrons, pions, and such. Perhaps the most important basic question that remains about the structure of the SME is the problem of relating the operators at the hadron level to quark-level operators and thus translating constraints resulting from experiments performed on real hadrons to bounds on the underlying quark parameters. We will use χ PT to begin addressing this question.

Of course, there could also be Lorentz violation in the gluon sector of the mSME. There are operators in the pure $SU(3)_c$ gauge sector with the same symmetries as the coefficients we shall be considering in this paper. They could contribute to some of the same hadronic coefficients as the quark coefficients listed below. So these gauge-sector parameters may be an important object of future study.

Moreover, there are also additional quark operators, besides those considered in this paper. We shall be looking solely at operators with mass dimension 4. These operators, because their Lorentz structures are similar to the structure of the conventional kinetic term for Dirac fermions, give rise to a rather complicated set of phenomena. Mixing between the conventional kinetic term and the dimension-4 Lorentz violation can give rise to a number of effects that have no analogues in models with other kinds of Lorentz-violating operators. So Lorentz-violating operators of mass dimension 3, although we will not be considering them in this paper, are expected to have a more limited array of physical effects; analyzing them will be the subject of future work.

This leaves the portion of the SME Lagrange density relevant to our present work, given

by [2]

$$\mathcal{L}_{\text{quark}}^{\text{CPT-even}} = i(c_Q)_{\mu\nu AB} \bar{Q}_A \gamma^\mu D^\nu Q_B + i(c_U)_{\mu\nu AB} \bar{U}_A \gamma^\mu D^\nu U_B + i(c_D)_{\mu\nu AB} \bar{D}_A \gamma^\mu D^\nu D_B. \quad (1)$$

The covariant derivatives contain the standard model gauge fields, and in curved spacetime the derivatives must be taken as linear combinations of derivative operators acting to the right and left. The left- and right-handed quark multiplets are denoted by

$$Q_A = \begin{bmatrix} u_A \\ d_A \end{bmatrix}_L \quad U_A = [u_A]_R \quad D_A = [d_A]_R, \quad (2)$$

and $A, B = 1, 2, 3$ label the quark generations. The $c_{\mu\nu}$ parameters in eq. (1) are dimensionless coupling coefficients that are Hermitian in the quark generation space spanned by A and B , while μ and ν are spacetime indices. Restricting the Lagrange density of Eq. (1) to up (u) and down (d) quarks, we may rewrite it as⁴

$$\mathcal{L}_{\text{light quarks}}^{\text{CPT-even}} = i\bar{Q}_L C_{L\mu\nu} \gamma^\mu D^\nu Q_L + i\bar{Q}_R C_{R\mu\nu} \gamma^\mu D^\nu Q_R, \quad (3)$$

where now $Q_{L/R} = [u_{L/R}, d_{L/R}]^T$ and the couplings are collected in the matrices

$$C_{L/R}^{\mu\nu} = \begin{bmatrix} c_{u_{L/R}}^{\mu\nu} & 0 \\ 0 & c_{d_{L/R}}^{\mu\nu} \end{bmatrix}. \quad (4)$$

Note that this formalism allows for there to be different $c^{\mu\nu}$ coefficients for the left-handed u and d quarks. Physically, the $SU(2)_L$ gauge invariance of the mSME requires that the coefficients for these two chiral fermion species be identical. Moreover, the separate coefficients for left- and right-handed chiral fermions are not typically what are observed in experiments with baryons. Experimental constraints are typically placed on the combinations $c^{\mu\nu} = \frac{1}{2}(c_L^{\mu\nu} + c_R^{\mu\nu})$ and $d^{\mu\nu} = \frac{1}{2}(c_L^{\mu\nu} - c_R^{\mu\nu})$.

It may also be convenient to split the coefficients into isosinglet and isotriplet pieces. These are ${}^1C_{L/R}^{\mu\nu} = \text{Tr}(C_{L/R}^{\mu\nu})$ and ${}^3C_{L/R}^{\mu\nu} = C_{L/R}^{\mu\nu} - (\mathbb{1}/2) {}^1C_{L/R}^{\mu\nu}$ (where $\mathbb{1}$ is the identity in flavor space).

It will frequently be important that the portions of these Lorentz-violating two-index tensors that are antisymmetric in their Lorentz indices cannot be observed at linear order. Only at second order in the Lorentz violation do these antisymmetric combinations have physical effects. This is a consequence of the fact that field redefinitions such as $Q'_L = [1 - (i/2)(C_L^{\mu\nu} - C_L^{\nu\mu})\sigma_{\mu\nu}]Q_L$ can actually eliminate the antisymmetric terms from the Lagrange density at first order [10]. In addition, as discussed below, in the absence of external gauge fields the antisymmetric terms do not contribute to the effective hadronic Lagrange density at leading order in the chiral power counting. We will thus assume $C_{L/R}^{\mu\nu}$ to be symmetric in the following.

⁴For a more complete analysis, one should explicitly integrate out the heavy degrees of freedom via the renormalization group.

Naively constructed Lagrangian terms involving the antisymmetric parts of the $C_{L/R}^{\mu\nu}$ tensors would also be odd under charge conjugation—depending on such left-right differences as $c_{u_L}^{\mu\nu} - c_{u_R}^{\mu\nu}$. Such C-odd terms cannot exist in the purely hadronic sector; they require the existence of additional external fields, which we are not considering here. However, the existence of C-odd bosonic terms that are antisymmetric in Lorentz indices and do explicitly involve external gauge fields was previously noted in [11]. We suspect that they may be a ubiquitous feature of Lorentz-violating theories with spin-0 excitations and external fields. However, very little is understood about these inherently Lorentz-violating terms.

Many of the best experimental bounds on Lorentz violation for nucleons come from atomic clock experiments involving nuclear magnetic transitions, and these are typically analyzed using only effective proton and neutron $c^{\mu\nu}$ and $d^{\mu\nu}$ coefficients. Yet there should also be additional couplings (analogous to the hadrons’ anomalous magnetic moments in the usual Lorentz-invariant theory) that appear in the Lorentz-violating effective action for hadrons interacting with an external electromagnetic field. The omission of these terms from existing analyses means that many quoted bounds should really only be interpreted as order of magnitude constraints. While we shall not consider the hadronic sector coupled to external fields in this paper, our χ PT methodology may easily be adapted to study such terms in the future.

3 Lorentz-Violating Hadronic Lagrangian

With the quark-level Lagrange density established, we are in a position to construct the effective Lagrangian at the hadronic level. This is done by considering all terms allowed by the symmetries of the underlying theory [4, 5, 6]. For quantum chromodynamics (QCD), these symmetries include the discrete operations C, P, and T. In addition, QCD possesses an accidental chiral symmetry in the limit of vanishing quark masses. Physically, the masses of the u and d quarks are much smaller than the masses of typical hadrons, which means that setting $m_u = m_d = 0$ is a reasonable starting point for the construction of the effective Lagrangian. Further it is known that the resulting $SU(2)_L \times SU(2)_R$ symmetry is spontaneously broken to the diagonal group $SU(2)_V$, and the pions are considered the associated Goldstone bosons. The corresponding pion fields are collected in the $SU(2)$ matrix [12]

$$U(x) = \exp \left[i \frac{\phi(x)}{F} \right], \quad (5)$$

where $\phi = \sum \phi_a \tau_a$ in terms of the $SU(2)$ generators, and $F \approx 92.4 \text{ MeV}$ is the pion decay constant in the $SU(2)$ chiral limit. Under chiral transformations $U(x)$ transforms according to

$$U(x) \rightarrow U'(x) = R U(x) L^\dagger, \quad (6)$$

where $(L, R) \in SU(2)_L \times SU(2)_R$. The mesonic effective Lagrangian in the chiral limit without the coupling to external fields can then be constructed in terms of $U(x)$ and its derivatives. At low energies the χ PT power counting dictates that derivatives acting on the

pion fields are suppressed, and the effective Lagrangian can be organized in terms of the number of derivatives. The leading order term is given by⁵

$$\mathcal{L}_\pi^{\text{LO}} = \frac{F^2}{4} \text{Tr}(\partial_\mu U \partial^\mu U^\dagger), \quad (7)$$

with the trace Tr being taken over flavor space.

In addition to spontaneous symmetry breaking, chiral symmetry is also explicitly broken by the non-zero quark masses. The mass terms for the light u and d quarks may be written as

$$\mathcal{L}_\mathcal{M} = -\bar{Q}_R \mathcal{M} Q_L - \bar{Q}_L \mathcal{M}^\dagger Q_R \quad (8)$$

with the quark mass matrix $\mathcal{M} = \text{diag}[m_u, m_d]$. Under chiral transformations of the right- and left-handed quark fields, $Q_R \rightarrow R Q_R$ and $Q_L \rightarrow L Q_L$, the mass term is not invariant. However, the pattern of symmetry breaking can be matched onto the effective chiral Lagrangian by assuming that the mass matrix transforms as $\mathcal{M} \rightarrow R \mathcal{M} L^\dagger$. The lowest-order chirally invariant term that is also even under C and P is then given by⁶

$$\mathcal{L}_{\text{s.b.}}^{\text{LO}} = \text{Tr}(\mathcal{M} U^\dagger + U \mathcal{M}^\dagger). \quad (9)$$

This term contributes at the same chiral order as the term in eq. (7).

Nucleon fields can also be considered. Under chiral transformations the nucleon doublet $\Psi = [p, n]^T$ transforms as [12, 13, 14]

$$\Psi \rightarrow K(L, R, U) \Psi. \quad (10)$$

The $SU(2)$ -valued function $K(L, R, U)$ is defined by

$$u(x) \rightarrow u'(x) = \sqrt{R U L^\dagger} \equiv R u K^\dagger(L, R, U) = K u L^\dagger, \quad (11)$$

where $[u(x)]^2 = U(x)$. Because K depends on the pion fields through $U(x)$, the covariant derivative of the nucleon field,

$$D_\mu \Psi = (\partial_\mu + \Gamma_\mu) \Psi, \quad (12)$$

contains pion fields in the chiral connection Γ_μ [15]. Recall that we have neglected the coupling of the nucleon field to external gauge fields, so the only term in the connection is

$$\Gamma_\mu = \frac{1}{2} (u^\dagger \partial_\mu u + u \partial_\mu u^\dagger). \quad (13)$$

The lowest-order pion-nucleon Lagrange density takes the form

$$\mathcal{L}_{\pi N}^{\text{LO}} = \bar{\Psi} \left(\not{D} - m + \frac{g_A}{2} \gamma^\mu \gamma_5 u_\mu \right) \Psi, \quad (14)$$

⁵We use \mathcal{L} to denote Lorentz-conserving and \mathcal{L} for Lorentz-violating Lagrange densities.

⁶The subscript “s.b.” refers to symmetry breaking.

where m is the nucleon mass, and g_A the axial vector coupling in the chiral limit. In the absence of external gauge fields

$$u_\mu = i(u^\dagger \partial_\mu u - u \partial_\mu u^\dagger). \quad (15)$$

In order to construct the effective Lagrangian including Lorentz violation in terms of hadronic degrees of freedom, we have to match symmetry properties of the quark-level expression from eq. (3) onto the hadronic level. In particular, under chiral transformations, $Q_R \rightarrow RQ_R$, $Q_L \rightarrow LQ_L$, the Lagrange density of eq. (3) transforms as

$$\mathcal{L}_{\text{light quarks}}^{\text{CPT-even}} \rightarrow i\bar{Q}_L L^\dagger C_{L\mu\nu} L \gamma^\mu D^\nu Q_L + i\bar{Q}_R R^\dagger C_{R\mu\nu} R \gamma^\mu D^\nu Q_R. \quad (16)$$

The matrices $C_{L/R}^{\mu\nu}$ are constant, and chiral symmetry is broken by the terms in eq. (3). Following the method described above for the quark mass terms in the QCD Lagrangian, we note that the Lorentz-violating action *would be* invariant under chiral transformations *if* $C_{L/R}^{\mu\nu}$ transformed as

$$C_L^{\mu\nu} \rightarrow L C_L^{\mu\nu} L^\dagger, \quad C_R^{\mu\nu} \rightarrow R C_R^{\mu\nu} R^\dagger. \quad (17)$$

Because of the cyclic property of the trace, this implies for the isosinglet and isotriplet components

$$\begin{aligned} {}^1C_L^{\mu\nu} &\rightarrow {}^1C_L^{\mu\nu}, & {}^3C_L^{\mu\nu} &\rightarrow L^3 C_L^{\mu\nu} L^\dagger, \\ {}^1C_R^{\mu\nu} &\rightarrow {}^1C_R^{\mu\nu}, & {}^3C_R^{\mu\nu} &\rightarrow R^3 C_R^{\mu\nu} R^\dagger. \end{aligned} \quad (18)$$

Using this transformation behavior to construct a Lagrange density that is invariant under chiral transformations, the pattern of symmetry breaking in the quark-level action can be matched onto the hadronic Lagrangian.

With these basic building blocks—and assuming the transformation behavior given by eq. (17)—we may construct the chirally invariant, Lorentz-violating leading order effective Lagrange densities for the pure pion sector and for pion-nucleon interactions. Writing down all possible expressions satisfying the symmetry properties produces some terms that are linearly dependent. By applying integration by parts and the symmetry properties of the $C_{\mu\nu}$ tensors, the number of terms may be reduced. Moreover, the transformation properties under parity and charge conjugation will also produce relationships among the various terms. The Lorentz-violating terms in the quark-level Lagrange density are the only potential sources of C , P , and T violations in this theory. So at leading order, the terms in the pion Lagrange density need to have the same discrete symmetries as the terms in the underlying quark density that are multiplied by the same $C_{L/R}^{\mu\nu}$ coefficients. This forces the coefficients for left- and right-handed quark fields to enter the pion Lagrange density multiplied by the same numerical low-energy couplings (LECs), drastically reducing the number of independent terms.

The LO minimal mesonic Lagrange density is given by

$$\mathcal{L}_\pi^{\text{LO}} = \beta^{(1)} \frac{F^2}{4} ({}^1C_{R\mu\nu} + {}^1C_{L\mu\nu}) \text{Tr}[(\partial^\mu U)^\dagger \partial^\nu U], \quad (19)$$

where $\beta^{(1)}$ is a dimensionless LEC. It encodes short-distance physics and cannot be determined from symmetry arguments. In principle, it could be calculated using nonperturbative QCD; however, the required calculations are not currently available. The factor of $F^2/4$ is present to mirror the form of the standard pion Lagrange density and is also chosen such that based on naive dimensional analysis [16] $\beta^{(1)}$ is expected to be of natural size, i.e. $\mathcal{O}(1)$.

Chiral symmetry also allows an analogous term for the isotriplet components ${}^3C_{R/L}^{\mu\nu}$ with an independent LEC,

$$\beta^{(2)} \frac{F^2}{4} \text{Tr}[(\partial^\mu U)^\dagger {}^3C_{R\mu\nu} \partial^\nu U + \partial^\mu U {}^3C_{L\mu\nu} (\partial^\nu U)^\dagger]. \quad (20)$$

However, this term can be shown to vanish because of the symmetry of ${}^3C_{R/L}^{\mu\nu}$ in the Lorentz indices.

In principle there is a second, nearly-identical-looking copy of the Lagrange density of eqs. (19) and (20) contracted with the antisymmetric parts of the $C_{L/R}^{\mu\nu}$. These terms would be accompanied by an independent set of LECs. However, all the terms involved can be shown to be total derivatives, so they may be dropped; and thus only the symmetric part of the $C_{L/R}^{\mu\nu}$ contributes at leading order.

Expanding $U(x)$ in terms of the pion fields shows that the Lagrange density in eq. (19) not only contains corrections to the pion propagator, but also induces new multi-pion interactions. The two-pion portion of the Lagrange density is

$$\mathcal{L}_\pi^{\text{LO},2\phi} = \frac{\beta^{(1)}}{2} (c_{u_L}^{\mu\nu} + c_{d_L}^{\mu\nu} + c_{u_R}^{\mu\nu} + c_{d_L}^{\mu\nu}) \partial_\mu \phi_a \partial_\nu \phi_a. \quad (21)$$

We shall defer most discussion of the term involving just two pion field operators until section 4, because such terms lead to the propagator modifications that have been used to constrain Lorentz violation in the pion sector.

For the moment, we shall concentrate on the forms taken by the pion vertices. Unfortunately, all three-pion vertices vanish when the symmetric parts of the $c_{L/R}^{\mu\nu}$ are involved. The four-pion vertex takes the form

$$\mathcal{L}_\pi^{\text{LO},4\phi} = \frac{\beta^{(1)}}{6F^2} (c_{u_L}^{\mu\nu} + c_{d_L}^{\mu\nu} + c_{u_R}^{\mu\nu} + c_{d_L}^{\mu\nu}) (\phi_a \phi_b \partial_\mu \phi_a \partial_\nu \phi_b - \phi_b \phi_b \partial_\mu \phi_a \partial_\nu \phi_a). \quad (22)$$

This term is a straightforward Lorentz-violating generalization of the usual four-pion vertex. Many Lorentz-violating operators in the SME Lagrange density are structurally similar to operators found in the usual standard model. For example, the quark kinetic terms from eq. (3) resemble standard kinetic terms, but instead of the indices on γ^μ and D^ν being contracted with the metric tensor $g_{\mu\nu}$, they are contracted with the Lorentz-violating backgrounds. The four-pion vertex can be similarly viewed as a deformation of the standard model four-pion vertex. Vertices with more pion fields can similarly be derived.

Looking at the pion sector overall, the two-pion term includes a $k^{\mu\nu}$ -type term that modifies the pion propagation. Terms of this type have previously been studied and experimentally constrained, and we shall discuss the physics of such terms in further detail

in section 4. The Lorentz-violating multi-pion interaction terms are new and have never been written down before, although the four-pion terms have a relatively straightforward structure that could probably be guessed at fairly easily.

However, it is an important observation that this vertex involves exactly the same $c_u^{\mu\nu}$ and $c_d^{\mu\nu}$ parameters that appear in the pion propagation Lagrangian. In the Lorentz-violating effective field theory approach, the Lorentz violation often has to be described using completely separate and independent coefficients multiplying the various operators. Understanding the relationships among these coefficients requires additional information—either about the physics underlying the Lorentz violation, or about the symmetry properties of the low-energy theory. For example, $U(1)$ gauge invariance can ensure that the same Lorentz violation coefficients appear in the kinetic term for a charged species and in the coupling to the electromagnetic field. In the pion theory, the chiral symmetry of the underlying physics provides concrete relationships between Lorentz-violating operators involving different numbers of fields. Since the two- and four-pion operators also involve the same LEC $\beta^{(1)}$, it would thus be possible to place constraints on purely interactional effects by looking at free particle propagation phenomena, and vice versa.

There are also relationships between Lorentz-violating behavior in the pure pion sector and Lorentz violation for baryons—for nucleons, in particular. The minimal LO baryonic Lagrange density is (recalling that Ψ is the nucleon doublet field)

$$\begin{aligned} \mathcal{L}_{\pi N}^{\text{LO}} = & \left\{ \alpha^{(1)} \bar{\Psi} [(u^\dagger {}^3C_R^{\mu\nu} u + u {}^3C_L^{\mu\nu} u^\dagger) (\gamma_\nu i D_\mu + \gamma_\mu i D_\nu)] \Psi \right. \\ & + \alpha^{(2)} ({}^1C_R^{\mu\nu} + {}^1C_L^{\mu\nu}) \bar{\Psi} (\gamma_\nu i D_\mu + \gamma_\mu i D_\nu) \Psi \\ & + \alpha^{(3)} \bar{\Psi} [(u^\dagger {}^3C_R^{\mu\nu} u - u {}^3C_L^{\mu\nu} u^\dagger) (\gamma_\nu \gamma^5 i D_\mu + \gamma_\mu \gamma^5 i D_\nu)] \Psi \\ & \left. + \alpha^{(4)} ({}^1C_R^{\mu\nu} - {}^1C_L^{\mu\nu}) \bar{\Psi} (\gamma_\nu \gamma^5 i D_\mu + \gamma_\mu \gamma^5 i D_\nu) \Psi \right\}, \end{aligned} \quad (23)$$

where the $\alpha^{(n)}$'s are dimensionless LECs that by naive dimensional analysis are expected to be $\mathcal{O}(1)$.

These four operators exhaust the possibilities at this order. Recall that these operators were formed by writing down all combinations of nucleon operators that would be chirally invariant if the Lorentz violation tensors transformed according to eq. (18). This essentially requires that the isotriplet components of the Lorentz violation tensors be sandwiched between u and u^\dagger to give $u^\dagger C_R^{\mu\nu} u$ and $u C_L^{\mu\nu} u^\dagger$.

There must also be two free Lorentz indices to be contracted with the $C_{L/R}^{\mu\nu}$ background tensors; and since we know in advance that only the operators that are symmetric in their Lorentz indices will contribute at leading order, it is advantageous to take the $C_{L/R}^{\mu\nu}$ to be symmetric from the start. This avoids the presence of terms such as those containing $[D_\mu, D_\nu]$. In addition, the antisymmetric combination of two nucleon covariant derivatives is of higher order in the χ PT power counting as well [17]. Note that $\mathcal{L}_{\pi N}^{\text{LO}}$ contains operators with the same two kinds of structures in spinor space as the underlying quark Lagrange density. These are, of course, not the only structures with two free Lorentz indices that may be constructed out of Dirac matrices and covariant derivatives. For example, the operator

could contain additional $D^\mu D_\mu$ or $\gamma^\mu D_\mu$ terms sandwiched between $\bar{\Psi}$ and Ψ . However, these terms can be eliminated using the equations of motion. For example, $i\gamma^\mu D_\mu \Psi = m_N \Psi$ (where m_N is the nucleon mass), up to higher-order chiral corrections; so at leading order, any term with an additional $\gamma^\mu D_\mu$ may be absorbed into one of the terms given in eq. (23). A dictionary of the possible reductions is given in [17].

There are really eight terms in eq. (23), arranged in pairs. As in the pion case, the discrete symmetries of the underlying quark theory force there to be specific relationships between the operators involving $C_L^{\mu\nu}$ and those with $C_R^{\mu\nu}$. There is no C violation in the chiral dynamics, so fermion operators that are even under C must be multiplied by likewise C-even combinations of $C_{L/R}^{\mu\nu}$ coefficients; this means symmetric sums of the corresponding coefficients for right- and left-handed quarks. Conversely, any C violation in the theory must be generated by C violation in the pattern of $C_{L/R}^{\mu\nu}$ coefficients. So a C-odd fermion operator must be multiplied by a difference between right- and left-chiral Lorentz violation coefficients. The well known transformation properties of Dirac bilinears indicate that the terms involving γ_5 are the ones that are odd under C. This accounts for the extra negative signs in the $\alpha^{(3)}$ and $\alpha^{(4)}$ terms; while eq. (3) with $C_L^{\mu\nu} = C_R^{\mu\nu}$ is even under C, with $C_L^{\mu\nu} = -C_R^{\mu\nu}$ it is odd.

As in the pion sector, there are two distinct ways that the Lorentz-violating tensors may enter. They may be traced over the flavor space, outside the Dirac bilinear, or they may be inserted between the two-flavor baryon spinor Ψ and its adjoint $\bar{\Psi}$. These two possibilities give different kinds of contributions to the separate Lorentz violation coefficients for protons and neutrons. The terms with traces sum over the u and d quark terms uniformly, giving the same contributions to the baryon Lorentz violation coefficients, regardless of isospin. However the terms with the $C_{L/R}^{\mu\nu}$ tensors actually contained within the fermion bilinear give quite different coefficients for the two nucleon species.

Finally, we point out that, because of the presence of the covariant derivatives in eq. (23), this nucleon Lagrange density actually contains interaction terms with arbitrary numbers of pions. These may be expanded directly, although we shall not consider them in further detail, because they are not presently useful for placing constraints on any Lorentz violation coefficients.

4 Experimental Constraints

We shall now turn our attention to setting new constraints on the effective Lorentz violation coefficients in one hadronic sector using experimental observations made in an entirely different sector. The key point is that there are only a limited number of underlying Lorentz violations for the quarks, which determine the effective coefficients for a much larger number of meson and baryon types. It is possible to measure or bound some combination of the quark coefficients using one type of hadron and transfer this information to another kind of particle entirely. The presence of numerous LECs limits the precision with which we may place bounds in the second particle sector, but there should still be order of magnitude validity, assuming the LECs have natural sizes. This will make our χ PT results very powerful.

So the forms we have found for the effective Lagrangians in the pion and nucleon sectors

have important physical consequences. We shall first return to the pure pion sector and look more closely at the propagation terms. Written in terms of the physical fields, the free two-pion Lagrange density is

$$\mathcal{L}_{2\pi}^{\text{LO}} = \frac{\beta^{(1)}}{2}(c_{u_L}^{\mu\nu} + c_{d_L}^{\mu\nu} + c_{u_R}^{\mu\nu} + c_{d_R}^{\mu\nu})(\partial^\mu \pi^+ \partial^\nu \pi^- + \partial^\mu \pi^- \partial^\nu \pi^+ + \partial^\mu \pi^0 \partial^\nu \pi^0). \quad (24)$$

This takes a standard form for Lorentz violation involving a spin-0 field. In the general Lagrange density

$$\mathcal{L}_{\text{spin-0}} = \frac{1}{2} \partial^\mu \phi_a \partial_\mu \phi_a + \frac{1}{2} k^{\mu\nu} \partial_\mu \phi_a \partial_\nu \phi_a - \frac{m^2}{2} \phi_a \phi_a, \quad (25)$$

the tensor $k^{\mu\nu}$ modifies the equations of motion—or, equivalently, the energy-momentum relation for free propagating particles. These propagation modifications can have many observable physical consequences.

Making very precise laboratory measurements with short-lived particles such as pions can be very challenging. As a result, most of the best constraints on Lorentz violation in the pion sector instead come from high-energy astrophysical observations [18, 19, 20]. When particles have modified energy-momentum relations, which do not have the standard relativistic forms, there may be upper and lower thresholds for various decay and emission processes. For example, with an appropriate choice of parameters, the decay of photons into charged particle-antiparticle pairs (such as $\gamma \rightarrow \pi^+ + \pi^-$) may occur for sufficiently energetic γ -rays. Observations of TeV γ -rays that have traversed astrophysical distances indicate that the threshold for this process, if it exists, must be above the energies of the measured photons, which means that particular combinations of the $k_\pi^{\mu\nu}$ coefficients must be correspondingly very small. A similar argument exists for another photon energy loss process that is ordinarily forbidden, $\gamma \rightarrow \gamma + \pi^0$. Other important processes are $\pi^0 \rightarrow \gamma + \gamma$, the normal π^0 decay mode, which could become disallowed above a certain energy, or $\pi^0 \rightarrow N + \bar{N}$, which would instead become the dominant decay mode if it were energetically allowed, because of the larger pion-nucleon coupling. Typically, astrophysical observations involving observed quanta at an energy E allow us to have constraints on combinations of $k_\pi^{\mu\nu}$ at the $\sim m_\pi^2/E^2$ level. In practice, this means there are bounds at the 10^{-10} – 10^{-13} levels, which are fairly strong. However, the bounds are on complicated combinations of all the $k_\pi^{\mu\nu}$ coefficients, which are determined by the sky coordinates of the sources involved. Moreover, there are much stronger bounds in other sectors, and there are limited possibilities for improving the direct pion bounds, since major improvements would require observations of substantially more energetic quanta, which can be few and far between.

According to eqs. (21) and (24) there is a single $k_\pi^{\mu\nu}$ tensor common to all the physical pion fields. The tensor takes the form

$$k_\pi^{\mu\nu} = \beta^{(1)}(c_{u_L}^{\mu\nu} + c_{u_R}^{\mu\nu} + c_{d_L}^{\mu\nu} + c_{d_R}^{\mu\nu}). \quad (26)$$

That the three pion types share these same leading order Lorentz violation coefficients should be no surprise, since in the chiral limit, the pion wave functions all contain equal mixtures

of the u and d fields, as well as equal right and left helicities. At this point, we should recall that physical $SU(2)_L$ gauge invariance requires that the underlying Lorentz violation coefficients for left-handed u and d quarks be the same, $c_{u_L}^{\mu\nu} = c_{d_L}^{\mu\nu} = c_{q_L}^{\mu\nu}$. This relation would be of crucial importance if we were seeking to relate the results of experiments performed on hadrons to the fundamental quark coefficients.

However, we shall instead focus here on the more concrete problem of relating separate sets of directly observable Lorentz violation coefficients. The key is that the combination $c_p^{\mu\nu} + c_n^{\mu\nu}$ of readily measurable baryon parameters depends on the exact same linear combination of quark parameters as the pion $k_\pi^{\mu\nu}$. The Lorentz-violating kinetic terms in the effective Lagrangian for the nucleon sector of the SME are written in terms of four coefficient tensors $c_p^{\mu\nu}$, $c_n^{\mu\nu}$, $d_p^{\mu\nu}$, and $d_n^{\mu\nu}$. These enter the Lagrange density for a species of Dirac fermions as

$$\mathcal{L}_{\text{spin}-\frac{1}{2}} = \bar{\psi} [i(\gamma^\mu + c^{\nu\mu}\gamma_\nu + d^{\nu\mu}\gamma_5\gamma_\nu)D_\mu - m] \psi. \quad (27)$$

(Additional dimension-three Lorentz-violating operators have been neglected.) That $c_p^{\mu\nu}$, $c_n^{\mu\nu}$, $d_p^{\mu\nu}$, and $d_n^{\mu\nu}$ number four should be no surprise, since that is also the number of independent tensors at the quark level, before $SU(2)_L$ gauge invariance is imposed.

In nonrelativistic experiments, $c_p^{\mu\nu}$ and $c_n^{\mu\nu}$ receive contributions from the $\alpha^{(1)}$ and $\alpha^{(2)}$ terms, while $d_p^{\mu\nu}$ and $d_n^{\mu\nu}$ receive contributions from the $\alpha^{(3)}$ and $\alpha^{(4)}$ terms. Starting from (27), the nonrelativistic Hamiltonian may be determined using a relatively straightforward Foldy-Wouthuysen transformation [21, 22], and most measurements of Lorentz violation involving protons and neutrons are done nonrelativistically, typically using atomic clocks [23, 24, 25, 26, 27, 28]. In this regime, it is possible to read off the effective coefficients directly from $\mathcal{L}_{\pi N}^{\text{LO}}$ with the pions neglected. For example, we obtain

$$c_p^{\mu\nu} = \left[\frac{1}{2}\alpha^{(1)} + \alpha^{(2)} \right] (c_{u_L}^{\mu\nu} + c_{u_R}^{\mu\nu}) + \left[-\frac{1}{2}\alpha^{(1)} + \alpha^{(2)} \right] (c_{d_L}^{\mu\nu} + c_{d_R}^{\mu\nu}). \quad (28)$$

One could attempt to use relations such as eq. (28) to place disentangled bounds on the u and d quark coefficients. However, the presence of the unknown LECs makes it impossible to do this fully quantitatively. The results would be rather unsurprising order of magnitude constraints on the quark-level parameters (which are not separately observable anyway).

It would be possible to proceed a bit further using a “quenched” approximation, under which the only quarks present in a nucleon are the valance quarks. In that case, the u quark contribution to the proton Lorentz violation should be twice the d contribution, and we may infer that $\alpha^{(1)} = \frac{2}{3}\alpha^{(2)}$. However, ignoring the presence of dynamical quark-antiquark pairs inside a nucleon is obviously a drastic approximation, and it is not clear how much physical value the $\alpha^{(1)} = \frac{2}{3}\alpha^{(2)}$ result has.

Summing eq. (28) and the analogous formula for neutrons gives an expression that is directly proportional to $(c_{u_L}^{\mu\nu} + c_{u_R}^{\mu\nu} + c_{d_L}^{\mu\nu} + c_{d_R}^{\mu\nu})$. Again, this is not particularly surprising. The resulting coefficients are effectively averaged over both spin and isospin, so they have equal contributions from all the quark tensors. Since this same combination of quark coefficients occurs in the pion $k_\pi^{\mu\nu}$, it is possible to place order-of-magnitude bounds on the $k_\pi^{\mu\nu}$ by combining observations made of the proton and neutron. Better than order-of-magnitude

Coefficient	Proton Bound	Neutron Bound
$c_Q = c_{XX} + c_{YY} - 2c_{ZZ}$	10^{-21}	10^{-10}
$c_- = c_{XX} - c_{YY}$	10^{-24}	10^{-28}
$c_{(XY)}$	10^{-24}	10^{-29}
$c_{(XZ)}$	10^{-25}	10^{-28}
$c_{(YZ)}$	10^{-25}	10^{-28}
$c_{(TX)}$	10^{-20}	10^{-5}
$c_{(TY)}$	10^{-20}	10^{-5}
$c_{(TZ)}$	10^{-20}	10^{-5}
c_{TT}	10^{-11}	10^{-11}

Table 1: Strengths of existing constraints on Lorentz violation in the proton and neutron sectors. Symmetrized combinations are denoted $c_{(\mu\nu)} = c_{\mu\nu} + c_{\nu\mu}$.

Coefficient	Bound
$(k_\pi)_- = (k_\pi)_{XX} - (k_\pi)_{YY}$	10^{-23}
$(k_\pi)_{(XY)}$	10^{-23}
$(k_\pi)_{(XZ)}$	10^{-24}
$(k_\pi)_{(YZ)}$	10^{-24}

Table 2: New constraints on pion Lorentz violation coming from comparisons to the nucleon sector.

accuracy is not possible, however, because of the presence of unknown LECs in all the hadronic expressions, which at the moment can only be estimated using naive dimensional analysis.

Bounds on SME parameters are typically expressed in a system of Sun-centered celestial equatorial coordinates, with the Z -axis pointing along rotation axis of the Earth, and the X -axis pointing to the vernal equinox point on the celestial sphere. The Y -direction is determined by the right-hand rule, and time in these coordinates is denoted by T . This coordinate system is well suited for describing the results of many kinds of terrestrial experiments, particularly measurements of spatial anisotropy that rely on the daily rotation of the Earth. Table 1 shows the best current order of magnitude constraints for the $c_p^{\mu\nu}$ and $c_n^{\mu\nu}$ coefficients, although recent analyses based on more careful nuclear models actually suggest significant improvements over some of these constraints [29, 30].

It is evident from table 1 that there are much stronger constraints in both the proton and neutron sectors than for pions for four types of coefficients. Table 2 therefore quotes new bounds on four pion parameters. Since the LECs in the nucleon and meson sectors are unknown but expected to be of $\mathcal{O}(1)$, we have set the pion constraints to be one order of magnitude weaker than the looser of the contributing proton and neutron bounds. Yet this still makes improvements of at least ten orders of magnitude over direct astrophysical

constraints on the same parameters.

5 Conclusions and Outlook

The ten order of magnitude improvement in pion sector constraints is evidence of how effective the χ PT method can be. We have produced disentangled bounds on four Lorentz violation coefficients that affect pion propagation, without looking directly at any pions. These bounds are among our most important results.

However, the effective Lagrange densities we have derived are also quite important. In the pure pion sector, we have a systematic way of generating multi-pion vertices with definite relations between them imposed by chiral symmetry. The lowest-order Lagrangian in the nucleon sector has also been laid out, again showing new terms. In the future, this kind of analysis may lead to an understanding of Lorentz violation for spin-1 and spin- $\frac{3}{2}$ composite particles, which have never really been studied in any detail.

The hadronic terms we have discussed—the $k_\pi^{\mu\nu}$ for pions, and the $c^{\mu\nu}$ and $d^{\mu\nu}$ for nucleons—are among the most important coefficients in the SME. These kinds of terms typically grow in importance with increasing energy, leading to the kind of unconventional thresholds discussed above. This is one reason why we chose to work with these kinds of terms.

However, there are still many more steps to be taken, and we have necessarily begun our χ PT analysis by considering only a subset of the mSME terms that are likely to affect hadrons. The most fundamental omission has been that we have not considered any Lorentz violation in the $SU(3)_c$ gauge sector. The strong interaction sector of the SME includes another set of pure gauge interactions that could make a leading order contribution to the two-index hadronic tensors such as $k_\pi^{\mu\nu}$ or $c_p^{\mu\nu}$. [There are also other possible Lorentz-violating terms in the $SU(3)_c$ action which, for symmetry reasons, cannot contribute to the hadron coefficients we have considered.] A more complete analysis should include the effects of both gauge and quark Lorentz violation on hadronic fields.

There are also other forms of Lorentz violation (many of which are, additionally, forms of CPT violation) that may exist for composite hadrons. There are more quark-level operators, particularly those with mass dimension 3, that will contribute in entirely different ways to symmetry violations by mesons and baryons. Detailed consideration of these operators will be another important task for the future.

Moreover, there may also be additional terms describing possible Lorentz-violating interactions between hadrons and external fields. Although we did not consider any external fields in this paper, it is obvious that there may be new interaction terms that look like Lorentz-violating deformations of the conventional anomalous magnetic moment interaction. Anomalous moments of baryons are typically large, and they often play a crucial role in atomic clock experiments. So including their Lorentz-violating analogues may be equally crucial to understanding how to interpret atomic clock tests of isotropy and boost invariance. Our ultimate understanding of how to use χ PT to analyze Lorentz-violating theories will certainly need to include consideration of these questions regarding external fields.

The question of how to relate the underlying quark and gluon coefficients in the mSME to the coefficients for composite hadrons has been one of the most important remaining puzzles in Lorentz-violating effective field theory. We have looked at how quark-level operators translate into pion, proton, and neutron operators, using the apparatus of χ PT. This and other recent work [8] demonstrate the power of the χ PT technique. It has enabled us to place new bounds (improved over previous ones by ten orders of magnitude) on pion-sector Lorentz violation, without directly studying pions. And yet there is still much more remaining to be understood about the use of χ PT in the context of the SME.

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